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Influence of a downstream narrowing on the flow profile in a tube

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Abstract

The distance over which the upstream flow conditions in a tube are disturbed by a stenosis downstream, i.e. the outlet length, was investigated for Reynolds numbers in the range 210–2900. Two methods were used, the Navier–Stokes equations were solved with a computer and a physical model was constructed and maximal velocities were measured with an ultrasound Doppler system. The computer model showed that Re number does not influence the outlet length, varying the stenosis area from 25% to 90% has an effect. However, the outlet length remained small, below 70% of the diameter of the tube. The physical model confirmed for a 75% stenosis that the outlet length is small, this method set the limit at not more than 1.2 times the tube diameter.

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Keywords: Inlet length; Flow profile; Blood flow; Navier–Stokes; Doppler ultrasound

1. Introduction

Doppler ultrasound instruments are applied for investigating flow in blood vessels. To assess the correctness of the signal processing in such an instrument a flow Doppler test object (FDTO), mimicking blood flow in a blood vessel in tissue, can be used (Hoskins, 1994). To make these measurements meaningful, the velocity profile at the position of measurement should be well defined. Often a fully developed parabolic flow profile is assumed (Teirlinck et al., 1997). It is well known that flow entering a straight tube only gradually approaches the parabolic profile (Tietjens, 1957; Schlichting, 1960). Formulae for the inlet length L_{in} required to give a fully developed parabolic profile can be found in Tietjens (1957), Hughes and Brighton (1991) and McDonald (1974). The formula for laminar flow can be written as $L_{in} = CD Re$ with C having values between 0.03 and 0.065 depending on the criterion used for non-disturbed flow.

One could imagine that also upstream the end of a straight pipe, where diameter and/or direction of the flow changes, the flow pattern is disturbed. Remarkably enough, the textbooks mentioned above and also Kays (1966), Vennard and Street (1982), Fox and McDonald (1985), Prandtl et al. (1990) and Mott (1994) do not indicate the length over which these phenomena occur. Observations with water flowing in rivers indicate that with a stenosis downstream, disturbances of the flow patterns propagate over a much smaller distance than for an upstream stenosis, therefore generally the phenomena with a downstream stenosis are neglected. However, it is not self-evident that this property of flow in open channels applies also to flow in pipes.

Goal of this study was to facilitate the construction of a FDTO (cf. Teirlinck et al., 1997 and IEC 61685, 2001), by determining the importance of a downstream stenosis by assessing the distance over which the flow conditions in a tube are disturbed. We call this distance the outlet length L . We have considered the simplest case: a sudden contraction of the diameter. With simple (single channel) Doppler instruments it is easy to determine the maximal occurring velocity in a cross section. Therefore we used this parameter in our tests. Fluid parameters

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Nomenclature

a	factor of change in maximal fluid velocity: $v_{\text{disturbed flow}}/v_{\text{undisturbed flow}}$
A_1	cross section of the tube (cm^2)
A_2	cross section of the stenosed part (cm^2)
C	coefficient in the formula for L
D	tube diameter (mm)
L	outlet length (mm)
P	signal level of the received Doppler signal (dB)
Re	Reynolds number

v	maximal (observed) velocity in a cross section (m/s)
x	position along the length of the tube, distance to the outlet point (mm)
$x_{1/2}$	position where the power of the Doppler signal (in dB) is halfway
x_{10}	position where flow disturbance starts, measured at a power of 10%
x_{25}	position where flow disturbance starts, measured at a power of 25%
η	dynamic viscosity (mPas)

were chosen to approximate those of blood in large vessels.

2. Methods

2.1. General description

Two different studies have been performed, a numerical experiment to calculate flow patterns from the Navier–Stokes equations and a physical experiment to measure the velocity in a tube with an ultrasonic Doppler instrument. In both studies we have concentrated ourselves on the study of the maximal velocity occurring in a cross section, as a function of the distance to a sudden narrowing in the tube. The studies have been performed on a range of velocities, characterized by their Reynolds number (Re). Reynolds numbers refer to the undisturbed flow in the tube.

The outlet length is determined by the point where the ratio a of the velocity disturbed by the outlet stenosis to the undisturbed velocity in the uniform tube $a = 1.05$. In case the uncertainty in the measurements does not allow the determination of this point, the point is used at which a significant change in velocity occurs.

In both studies we have used a long straight tube to establish a well-defined flow pattern. At the end of the tube the diameter is axisymmetrically reduced (see Fig. 1). We have studied changes in the axial velocity as a function of the distance x upstream of the narrowing. We assume that the axial velocity is the highest velocity occurring in a cross section of the tube.

2.2. Numerical study

The time-dependent incompressible Navier–Stokes equations for an axisymmetric flow were numerically solved; details of the numerical algorithm can be found in Verstappen and Veldman (2003). In all calculations laminar flow was assumed. A tube was simulated with a

length of 2000 mm and an internal diameter of 8 mm. At 1200 mm downstream of the entrance, the diameter was abruptly decreased to 4 mm, this is a stenosis of 75%. The calculating mesh had an axial resolution of 0.3 mm and a radial resolution of 0.3 mm. These resolutions are sufficient to limit deviations from the incoming parabolic flow profile to 0.01%. In the contraction region the numerical discretisation error was found to be well under 1%, which is an acceptable error in this study. The fluid had a dynamic viscosity $\eta = 4$ mPas, and a density of 1000 kg/m^3 . Calculations have been made for values of $Re = 210, 1250, 2000$ and 2900 .

A uniform velocity profile has been chosen as input condition, the velocity v was derived from Re . The flow condition in the tube was simulated in time until no further changes in axial velocity were observed. The sensitivity of the value of the outlet length for the value of the parameter a has been determined in the numerical study.

In addition to the above-described situation the severity of the stenosis was varied.

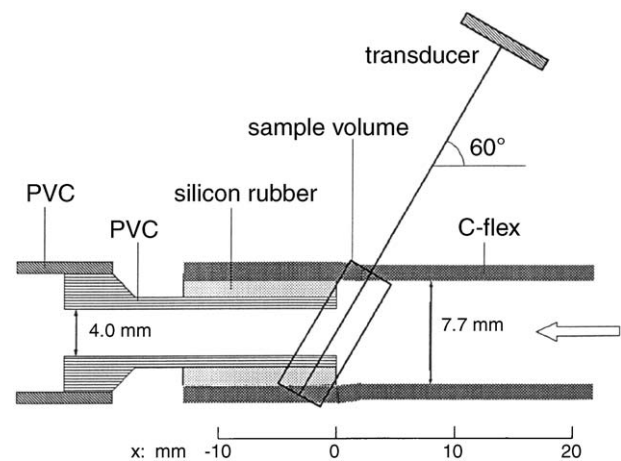


Fig. 1. Cross section of the outlet part in the ultrasound experiment. The figure is to scale. The arrow indicates the flow direction for the case of downstream stenosis. The position of the active element of the transducer and the sample volume are indicated for the position $x = 0$.

2.3. Ultrasound experiment

A flow circuit was formed using a reservoir, a centrifugal pump and a C-flex tube (Cole Parmer, E-06424-75, Vernon Hills, IL, USA) of nominal inner diameter of 8.0 mm and wall thickness of 1.6 mm. At the inlet side of this tube a 90° bend was present with a diameter of 8.5 mm. At the end of the C-flex tube a PVC pipe was inserted with an inner diameter of 4.0 mm, area $A_2 = 0.126 \text{ cm}^2$ (see Fig. 1) and connected to the C-tube via a silicone rubber bush. The length between the 90° bend and the beginning of the narrow section was 334 mm.

The C-flex tube was put under slight tension (longitudinal strain 1.7%) to straighten the tube. The measured area from a tube of the same supply appeared to be 0.474 cm^2 , the tension reduces the area by 1.7% to $A_1 = 0.466 \text{ cm}^2$, resulting in a reduction factor $A_1/A_2 = 3.7$. The fluid was similar to the blood mimicking fluid (BMF) described by Ramnarine et al. (1998) containing Orgasol particles (5 µm) as scattering particles. We used a slightly modified fluid (BMFa, see Lubbers, 1999) to improve the durability of the BMF by adding sodium azide. Dynamic viscosity was found to be 3.8 mPas, density 1035 kg/m^3 .

The fluid was sieved by an in-line filter (Millipore filterholder XX4304700, Millipore, Bedford, USA) with a 50 µm mesh filter (Monodur® polyamide fabric PAM.N.50.102, Verseidag-Techfab Geldern, Walbeck, Germany). This was found to be useful in removing debris in the flow circuit, which showed up as spikes in the Doppler spectrum. The fluid was repeatedly degassed by exposing it to a pressure just above the vapour pressure of water.

The BMF was pumped through the tube at a fixed rate. Average flow velocity was calculated from tube cross section and readings of the flow meters. (Note: in agreement with McDonald (1974) we use the term ‘average’ velocity in the sense of average over a cross section at any moment, and the term ‘mean’ velocity for the mean of time-dependent variations.)

The flow meters were calibrated with the BMF described by Ramnarine et al. (1998) using a graduated vessel and a stopwatch. Readings of flow meters during an experiment varied at the most by $\pm 1.5\%$.

The ultrasonic observations were carried out with a pulsed Doppler system (TC2-64B, EME, Eden Medizinische Technik, Ueberlingen, Germany), equipped with the ECP4 transducer, an unfocused 4.00 MHz pencil probe with a crystal of 8 mm diameter, and a natural focal depth of 32 mm as measured in a water bath (all data according to the manufacturer). This Doppler instrument displays the sonogram with a 3-level gray scale.

The Doppler system calculated the maximum observed velocity, taking the mean value over an observa-

tion screen (duration 2 s). The resolution of this reading was 2 cm/s. These values were read from the on-screen indication. The reading is influenced by the settings of the instrument. We used a fixed receiver gain (at mid-range), and two settings of the emitted power (10 and 25% of full power). Lower power was not available, the next higher power step (50%) caused spurious detections of noise components. In the absence of significant noise our Doppler indicates the correct velocity for signals above a certain level, for signals of less power, a too low value is indicated. The level of the Doppler signal was measured with a logarithmic amplifier (Hewlett Packard 7562A, Pasadena, CA, USA) with the high pass filter set at 50 Hz. Signal levels are quoted in dB above the noise level.

The axial window of the instrument was measured with a line target (1 mm tube, carrying a flow). It was found that over 10 mm length in the axial direction the window had a constant sensitivity, with a steep drop outside this ($> 20 \text{ dB}$ over 2 mm). The window can be positioned in steps of 2.5 mm.

The C-flex tube was mounted in a water bath. The centre of the tube was 10 mm above the bottom. The Doppler probe was kept under an angle of 60° with the tube axis. The observation window was centred on the tube in the axial direction of the probe. The midpoint between the two settings where the signal-level changes much due to loss of flow information from the window was taken as the best axial alignment. The centre of the window was positioned at 27.5 mm from the transducer. (This is slightly nearer to the probe than the natural focal depth of the probe, being 32 mm. In view of the weak focusing of the probe, this is acceptable.) Perpendicular to the tube the probe was centred by eye.

The tube was scanned parallel to the tube axis. The position of the probe was measured with a ruler. Generally, the scans around the transition between the wide and the narrow tube were made at every 2 mm. The accuracy of the positioning was estimated to be 1 mm. For each measured point we noted the signal level P (in dB) and maximal velocity v in the Doppler spectrum. The Doppler instrument renders the observed velocity, i.e. the component of the velocity along the axis of the probe. As the Doppler angle $\theta = 60^\circ$ (see Fig. 1) true velocities are twice as large ($1/\cos \theta$) as the observed velocities. All results are given as true velocities. The resolution of the velocity scale is 0.04 m/s. Scans were performed for $Re = 210, 1250, 2000$ and 2900.

2.4. Validation

The constructed stenosis has two features which might influence the results obtained with the Doppler instrument. In the stenosis the ultrasound has to transverse a much thicker wall than in the C-flex tube. Thus the reflected ultrasound is much more attenuated

for signals from the inside of the stenosis. Further the wall echos might be different in both situations. It is well known that strong stationary echos can disturb the processing of the weak signals from moving blood (IEC 61206, 1993, p. 23, 24).

To validate the ultrasound set-up, we have reversed the flow direction through the diameter change by building a flow commutator, thus creating an inlet situation, in which velocities have been determined. The continuity of results going from the stenosis to the wide tube can be used as an indication that no gross artefacts are present in the Doppler results.

Flow directions are indicated as D when the stenosis is downstream of the wide tube and as U when it is upstream of the wide tube.

3. Results

3.1. Numerical study

At the entrance of the tube a flat flow profile was applied. It was found that the axial velocity in the wide tube increases gradually downstream of the entrance, indicating the development of a parabolic flow profile. For $Re = 210$ the distances over which the axial velocity stabilized in a tube were very much shorter than at $Re = 2000$, fully in line with the dependence of the inlet length as discussed in the Introduction. In both cases a parabolic profile was established near the entrance of the stenosis. The region around the diameter transition is shown in detail in Fig. 2 for $Re = 210$ and 2000. At the narrowing a sudden jump to a high velocity is seen. It appears that the stenosis influences the axial velocity only over a few mm upstream. Taking as a perceptible disturbance, a relative change of the axial velocity $a = 1.05$, the disturbance starts at 3 mm, hence at a distance of less than half the tube diameter from the transition. Table 1 shows the influence of flow rate (characterized by Re) and of the choice of the parameter a on the outlet length.

Table 2 shows the influence of the degree of the stenosis for a flow with $Re = 210$ in the wide tube. It can be seen that the smaller the stenosis is, smaller is the distance over which the outlet effect occurs. If a large value of the criterion a is used, the criterion may only be met for the flow inside the stenosis (negative numbers in Table 2). Also for these stenoses the outlet effects do not start before one tube diameter (8 mm) from the end of the wide tube.

3.2. Ultrasound experiment

Typical results of the ultrasound experiment are shown in Fig. 3 (for the whole tube) and Fig. 4 (for the region around the narrowing) for $Re = 2000$. The

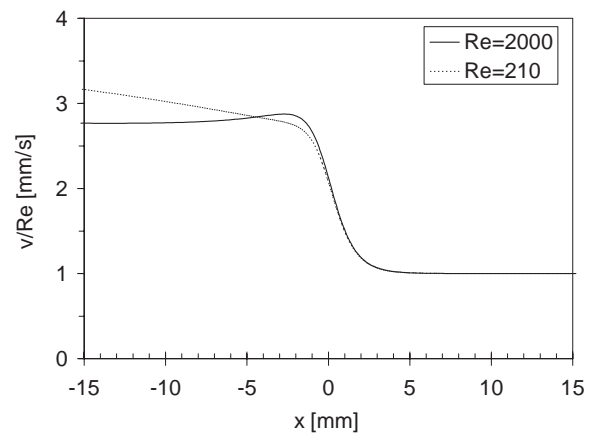


Fig. 2. Result of numerical study. The axial velocity v/Re (in mm/s) as a function of the position x (in mm) in the tube. At $x = 0$ mm the diameter changes from 8 to 4 mm. The flow is from the right to the left. The cases $Re = 210$ and 2000 are shown.

Table 1
Result of numerical study

	$Re = 210$	$Re = 1250$	$Re = 2000$	$Re = 2900$	average
a	L	L	L	L	L/D
1.01	4.87	4.89	4.89	4.89	0.61
1.05	3.31	3.30	3.30	3.29	0.41
1.10	2.65	2.63	2.62	2.62	0.33
1.20	1.98	1.95	1.95	1.95	0.24
1.50	1.06	1.03	1.03	1.02	0.13

The outlet length L (in mm) for various values of the relative change a of axial velocity that is used as criterion. Data are given for four values of Re ; the numerical error in L is a few units in the second decimal. Also the distance L normalised to the tube diameter D is given for the average value of L over the four Reynolds numbers.

Table 2
Result of numerical study

a	Stenosis			
	25%	50%	75%	90%
1.01	2.46	3.93	4.87	5.31
1.05	0.29	2.26	3.31	3.79
1.10	-1.67	1.47	2.65	3.16
1.20	-15.19	0.52	1.98	2.55
1.50	not found	-3.63	1.06	1.78

The influence of the degree of stenosis on the outlet length L (in mm) for various values of the relative change a of axial velocity that is used as criterion. Data are for $Re = 210$.

errors in velocities below 1 m/s are dominated by the rounding off by the Doppler system (± 0.02 m/s), above 1 m/s some fluctuations were seen. Sound levels were measured to the nearest dB, fluctuations of 1 dB are not significant. The following phenomena are observed in Fig. 3: with the system at an emitting power of 10% the signal is too weak to be detected inside the narrow part,

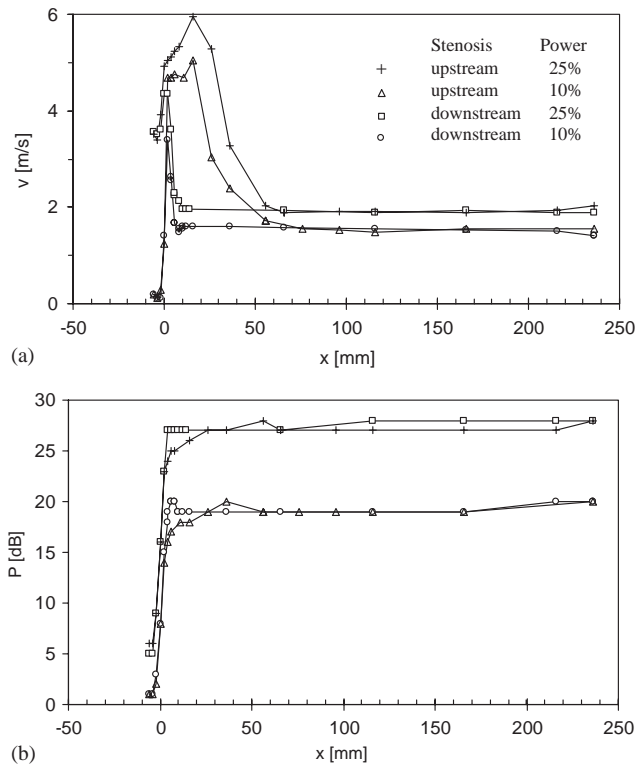


Fig. 3. Result of ultrasound experiment. (a) The maximal velocity v ($Re = 2000$) as a function of position along the tube. Results are shown for the flow directions with stenosis upstream and for stenosis downstream, as well as for two values of the intensity of the emitter, 10% and 25% of full power. (b) The difference P between the signal level and the noise level (in dB) as a function of position in the tube. Symbols as above.

the indicated velocity there is very small. In the wide part sufficient signal is present for meaningful data, although the indicated velocity is still slightly too low. At an emitting power of 25% the signal is stronger in the wide part, leading to slightly higher observed values for the axial velocity. In the narrow tube the 25% signal is too weak for full detection, but strong enough to show that high velocities are present.

With a downstream stenosis it can be seen that from higher to lower values of x the axial velocity is almost constant. The flow starts to increase from $x = 10$ to 6 mm (D25) or 8 to 4 mm (D10). Using the 5% criterion the outlet trajectory starts at 9.4 mm (D25) and 6.2 mm (D10). However, the scans were not fine enough to warrant such a detailed analysis, therefore we prefer to indicate an interval for the possible start of the outlet trajectory. We denote these results by $6 < x_{25} < 10$ mm, and $4 < x_{10} < 8$ mm.

The point where the narrow tube starts was determined from the geometry (measuring accuracy 1 mm). The signal level data provide a check on the positioning. The signal level drops by nearly 20 dB going from the wide to the narrow tube (this is mainly caused by

additional attenuation of the ultrasound when insonating the narrow tube, due to additional wall materials (silicone rubber and PVC)). As a characteristic point on the curve we take the point $x_{1/2}$ where half of the change in signal strength (in dB) between the narrow and the wide tube has occurred. It was found that $x_{1/2}$ generally lies between $x = 0$ and -1 mm (see Table 3). This table shows further that at other Re the results for x_{10} and x_{25} were very similar to the ones given above. Fig. 5 shows the results for $Re = 210$. Table 3 summarizes for the five used values of Re the obtained results, given as intervals of possible values, taking into account the resolution of the scans.

3.3. Validation

With an upstream stenosis high velocities are present over a distance of circa 70 mm at $Re = 2000$ (see Fig. 3). At $Re = 210$ (see Fig. 5) this region is even longer (100 mm). In this region there is also negative velocity (not shown in the figures). This indicates a jet stream from the stenosis, with sideways counter-current flows. After 70, resp 100 mm the jet stream fills the whole tube, leading to much lower peak velocities. Further downstream the axial velocity is almost constant.

4. Discussion

4.1. Numerical study

Fig. 2 shows that a downstream 75% stenosis influences the flow pattern over a very short distance. Table 1 shows that the influence of the flow rate is negligible from $Re = 210$ to 2900. Using the criterion $a = 1.05$ the influence extends over a distance of 3 mm, i.e. much less than the tube diameter. For reasonable values of the criterion $1.01 < a < 1.50$ the range of the influence is 1–5 mm.

Table 2 shows that for a more severe stenosis the influence reaches slightly further, but even with a stenosis of 90% it stays within one tube diameter (8 mm). Here we present only data for $Re = 210$, but we have confirmed that in line with Table 1 at other values of Re , the results are very similar.

4.2. Ultrasound experiment

When comparing x_{25} with x_{10} , x_{25} is generally 0 to 2 mm larger than x_{10} , caused by the Doppler system that will detect a high-frequency admixture to the signal easier at high than at low signal levels. When using the more sensitive x_{25} the point where the first increase in axial velocity is seen lies between 4 and 10 mm upstream of the stenosis (see Table 3).

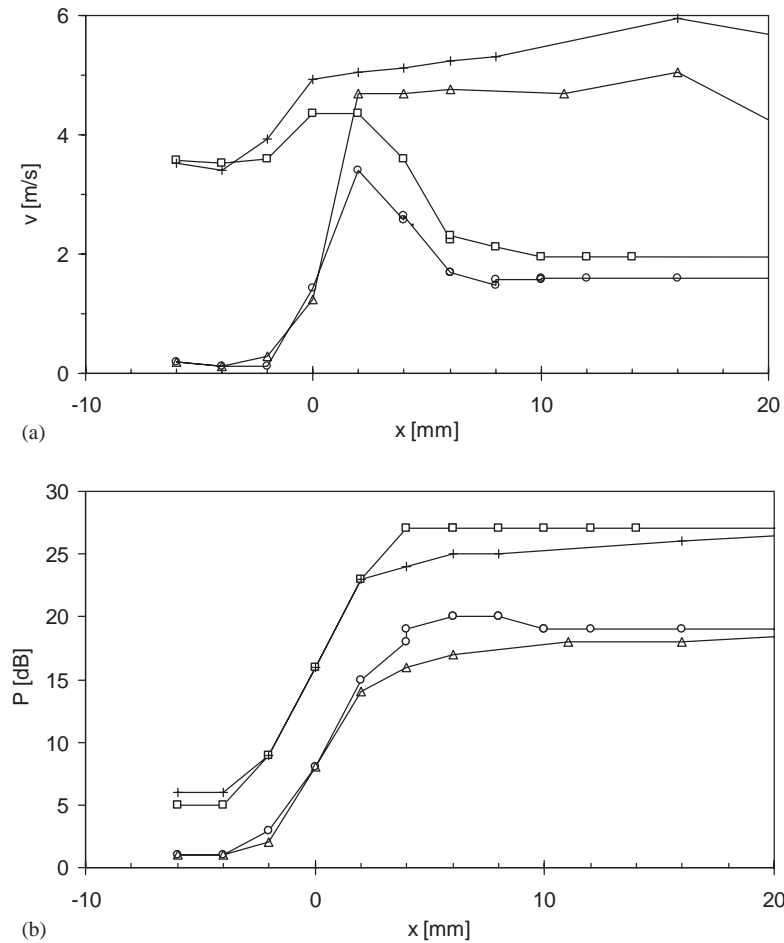


Fig. 4. Detail from Fig. 4. Legend as for Fig. 3.

Table 3
Result of ultrasound experiment

Re	$x_{1/2}$ (mm)	x_{10} (mm)	x_{25} (mm)
210	−0.5 to 0.5	4 to 6	4 to 6
1250	−1.3 to −0.7	4 to 8	4 to 8
2000	−0.3 to 0.7	4 to 8	6 to 10
2900	−1.0. to −0.2	3 to 6	4 to 6

Intervals for the half value position of the sound intensity ($x_{1/2}$) and the positions where the first increase in velocity was observed (x_{10} and x_{25}).

This position refers to the centre of the transducer. The ultrasonic transducer has a finite beam width. For an 8 mm transducer an effective beam diameter at the natural focus of 2 to 4 mm does not seem unreasonable. As high velocities at the border of the beam may be detected, the above data imply that high velocities start 1 to 2 mm nearer to the stenosis than the centre of the probe. Further the uncertainty in geometry (1 mm) should be taken into account. Hence the result from the Doppler measurements is that the value of the outlet length L is thus $1 < L < 9$ mm.

This statement applies to all used values for Re in the wide part of the tube, i.e. $200 < Re < 2900$. Within the experimental inaccuracy of x_{25} (range 2 to 4 mm for each data point) the result of the numerical study (no influence of flow rate on outlet length) is confirmed.

4.3. Validation

Signal intensities for upstream and downstream stenosis are generally equal along the tube. An exception has to be made near the stenosis. The explanation is that with an upstream stenosis a narrow jet is formed, aside of which fluid is present which flows so slowly that its Doppler frequency is below the threshold of the high pass filter (50 Hz). Thus effectively less fluid is observed by the Doppler system, leading to lowering of the signal level by 2 to 3 dB. This is particularly evident in Fig. 5. The equality of signal intensities in the inlet and outlet situations, in spite of the observed changes in velocity in the inlet situation, indicates that the Doppler equipment gives a valid indication of maximal velocity in the tube.

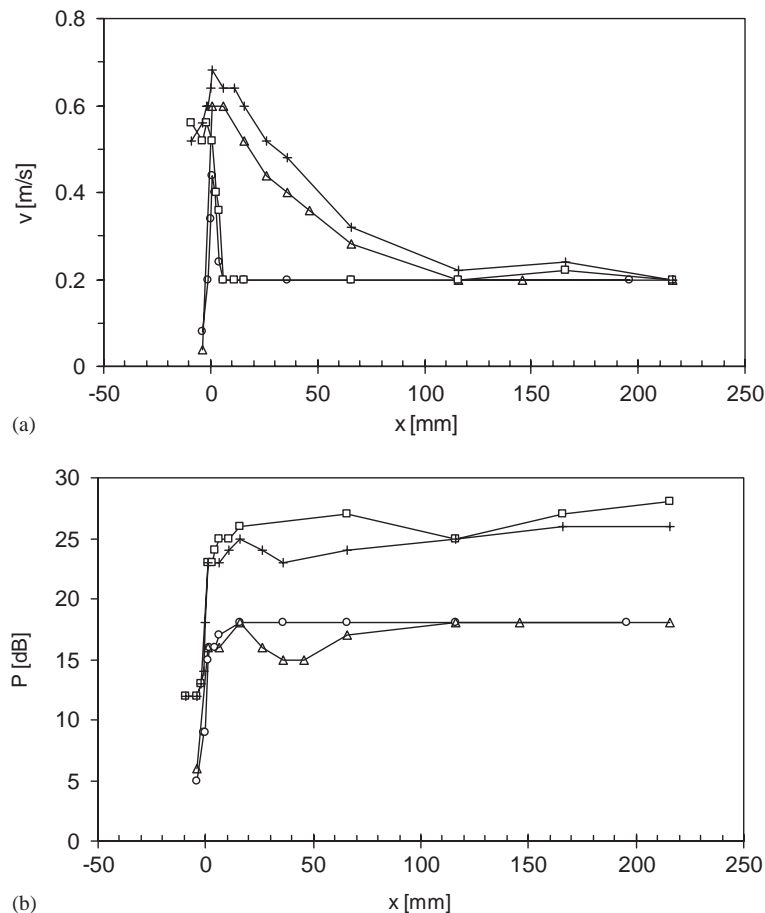


Fig. 5. Data for $Re = 210$. Legend as for Fig. 3.

4.4. General remarks and conclusion

The result of the experimental study is ($1 < L < 9$ mm). This result can be summarized as follows: for Re between 210 and 2900 the influence of a 4-mm-downstream axial stenosis influences the flow in an 8 mm tube over a distance between 1 and 9 mm. This can be generalized as: a 75% area axisymmetric stenosis influences the axial velocity not further than 1.2 diameters upstream.

The numerical results are much more precise. They indicate that over the range $Re = 210$ to 2900 and stenosis between 25% and 90% the entry length is not more than 0.7 tube diameters. In case an accurate experimental verification of the numerical study is desired, an experimental method with higher geometric resolution is needed, e.g. using light.

For application to the construction of a FDTO the results of both studies are similar. They can be summarized as follows: an outlet length of one tube diameter before a stenosis is sufficient to prevent disturbance of the flow profile in the wide tube.

Our result is only valid for a stenosis which is placed axisymmetric. Asymmetric stenosis may have larger

influences. Another situation which may influence the flow profile is a downstream bend. Working with tubing, this will always mean a change in diameter, either by bending the tube itself or using a connection piece. This bend will be further downstream than the diameter change, so it is unlikely that the bend will have more influence than the diameter change.

References

- Fox, R.W., McDonald, A.T., 1985. Introduction to Fluid Mechanics. Wiley, New York, pp. 366–367.
- Hoskins, P.R., 1994. Review of the design and use of flow phantoms. In: Hoskins, P.R., Sherriff, S.B., Evans, J.A. (Eds.), Testing of Doppler ultrasound equipment. Report 70. The Institute of Physical Sciences in Medicine, York, UK.
- Hughes, W.F., Brighton, J.A., 1991. Theory and Problems of Fluid Dynamics, Second ed. Schaum's outline series, McGraw-Hill, New York.
- IEC 61206, 1993. Ultrasonics—Continuous wave Doppler systems—test procedures. IEC (International Electrotechnical Commission), Genève, Suisse.
- IEC 61685, 2001. Ultrasonics—flow measurement systems—flow test object. IEC (International Electrotechnical Commission), Genève, Suisse.

- Kays, W.M., 1966. *Convective Heat and Mass Transfer*. McGraw-Hill, New York.
- Lubbers, J., 1999. Application of a new blood-mimicking fluid in a flow Doppler test object. *Eur. J. Ultrasound* 9, 267–276.
- McDonald, D.A., 1974. *Blood Flow in Arteries*, Second ed. Edward Arnold, London.
- Mott, R.L., 1994. *Applied Fluid Mechanics*, Fourth ed. Merrill (Macmillan Publ Cie), New York, pp. 267–279.
- Prandtl, L., Oswatitch, K., Wieghardt, K., 1990. *Führer durch die Strömungslehre*, Ninth ed. Vieweg, Braunschweig.
- Ramnarine, K.V., Nassiri, D.K., Hoskins, P.R., Lubbers, J., 1998. Validation of a new blood-mimicking fluid for use in Doppler flow test objects. *Ultrasound in Medicine and Biology* 24, 451–459.
- Schlichting, H., 1960. *Boundary layer theory*, Fourth ed. (translated by Kestin J.), McGraw-Hill, New York.
- Teirlinck, C.J.P.M., Fish, P., Hoskins, P.R., Lubbers, J., Fredfeldt, K.E., Kollmann, C., Schaarschmidt, U.G., Zeqiri, B., Nassiri, D., 1997. Validation of a flow Doppler test object for diagnostic ultrasound scanners, final report contract MAT1-CT 940091 (European Community). Report no TG/97.005, TNO Leiden, Leiden.
- Tietjens, O.G., 1934 (unaltered reproduction 1957). *Applied hydro- and aeromechanics*, based on lectures of Prandtl L. (translated by Den Hertog, J.P.), Dover publications, New York.
- Vennard, J.K., Street, R.L., 1982. *Elementary fluid mechanics*. Sixth ed. (SI version). Wiley, New York.
- Verstappen, R.W.C.P., Veldman, A.E.P., 2003. Symmetry-preserving discretization of turbulent flow. *Journal of Computational Physics* 187, 343–368.